Journal of Sound and Vibration (1998) **213**(1), 75–88

FREE VIBRATION OF CLAMPED–FREE CIRCULAR CYLINDRICAL SHELL WITH A PLATE ATTACHED AT AN ARBITRARY AXIAL POSITION

J. S. Yim and D. S. Sohn

Department of Advanced Fuel Development, Korea Atomic Energy Research Institute, Zip 305-353, *PO Box* 105, *Yusong, Taejon, Korea*

and

Y. S. Lee

Department of Mechanical Design Engineering, Chung-Nam National University, Zip 305-764, *Kung-Dong, Yusong, Taejon, Korea*

(*Received* 12 *March* 1997, *and in final form* 12 *December* 1997)

In this paper, analytic formulations and their results are presented to extend the receptance method to a clamped–free cylindrical shell with a plate attached in the shell at an arbitrary axial position. Prior to the analysis of the combined system, the analysis of the free vibration for the shell without a plate was performed by the Rayleigh–Ritz method with a beam function. The integration of the beam functions was performed symbolically by Mathematica and was incorporated in a solution program that could be run in a personal computer. After getting the eigensolution of the simply supported circular plate, the frequency equation of the combined system was obtained by considering the continuity condition at the shell/plate joint. The numerical results were compared with the results from ANSYS, as well as a free vibration test, to validate the formulation. The comparison showed that the analytic results agreed with those from ANSYS and the test.

1. INTRODUCTION

Many analyses of the free vibration of clamped–free cylindrical shells by means of the Rayleigh–Ritz method have been presented in the literature [1–5]. The Rayleigh–Ritz method was commonly used when the general solution was either hard to find or very cumbersome to manipulate. This method yields an exact frequency equation when the exact modal functions, which satisfy the geometric as well as the natural boundary conditions of a given problem, are chosen [6]. A beam function has been widely used as the modal function of a cylindrical shell in the axial direction. When a beam function is assumed to be a modal function of the shell, the integration of the function along the shell length, which is quite difficult, is required. However, owing to the brilliant development of packages for manipulating functions, such as Mathematica [7], it has been no longer hard to manage these functions. Yim *et al*. [8] presented the analysis of a cantilevered shell with a beam function where it was shown how to get the cumbersome integration of the beam functions without any difficulties.

The receptance method was introduced by Shkarov [9] for combined structures. Huang and Soedel [10, 11] presented the results of an analysis of the free vibration of both ends of a simply supported cylindrical shell with a plate at an arbitrary axial position using the receptance method. Since the boundary condition of the problem was the simple supported

0022–460X/98/210075 + 14 $25.00/0/sv971498

condition at both ends, a symmetric simple trigonometric function, i.e., a sine function, was assumed as the modal function.

If a plate is attached in a clamped–free cylindrical shell at an arbitrary axial position, such as a storage tank or silo with a flat circular plate, this combined structure may be analyzed by various methods. Suzuki *et al*. [12] analyzed the free vibration of a cantilevered shell with a circular plate attached at the top by applying Mindlin theory to the plate. Irie *et al*. [13] presented an analyses of the free vibration of a joined conical-cylindrical shell by way of the transfer matrix technique. Tavakoli and Singh [14] presented the eigensolution of joined/hermitic shell structures using the state space method.

Of course, the finite element method may be another powerful solution method for the combined structures. However, the preparation of input for the solution still takes much time and the post-process to interpret the solution is also difficult. The commercial FEM package ANSYS [15] calculates the natural frequencies rather easily due to the improvement of the preprocessor in making constructive input. Nevertheless, to identify the mode shapes corresponding to certain frequencies, the solution must be extracted for each mode shape and classified one by one, which can be very tiresome work. If an analytic solution for a structure is given, it yields accurate natural frequencies and mode shapes very fast and easily.

The frequencies and mode shapes from the analytic method were compared with the results from ANSYS, as well as a free vibration test, to validate the formulation. The comparison showed that the analytic results agreed well with those from ANSYS and the test.

2. THEORETICAL FORMULATION

2.1. DISPLACEMENT OF THE SHELL DUE TO DYNAMIC LOADING

The analysis of the free vibration of a shell/plate combined system can be performed by the Receptance method. The term Receptance is defined by the ratio of the response of a structure to the input function. Thus, if the input forcing function is defined, the response of the system and the receptances can be obtained. Once the receptances are calculated, the frequency equation can be derived by considering the continuity condition at the joint. The input forcing function of the shell/plate combined system will be the force or moment at the joint produced by the constraint of the motion of shell by the plate or vice versa.

Figure 1 represents the schematic view of a clamped–free cylindrical shell with a plate attached at the top of the shell. Figure 2 shows the cross-sectional view of the displacements and the slopes at joining points due to the dynamic transverse line loads and line moments around the shell exerted by the motion of vibration.

The displacements of the shell subjected to a dynamic loading at the junction can be expressed by the modal displacement and mode participation factor [16] as

$$u_i(x, \theta, t) = \sum_{k=1}^{\infty} \eta_k(t) U_{ik}(x, \theta), \qquad i = 1, 2, 3, \tag{1}$$

where k designates the mode number, u_i represents the displacement of the shell in the axial direction ($i = 1$), circumferential direction ($i = 2$) and normal to the surface ($i = 3$).

In equation (1), the mode participation factor η_k is the root of the following modal equation for the steady state harmonic response of the shell:

$$\ddot{\eta}_k + 2\zeta_k \omega_k \dot{\eta}_k + \omega_k^2 \eta_k = f_k \, e^{j\omega t}, \tag{2}$$

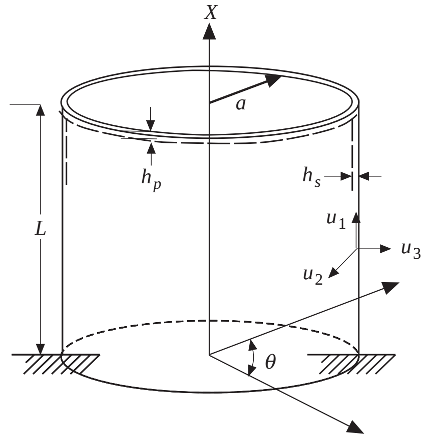

Figure 1. Co-ordinate system and notation.

where

$$f_k = 1/\rho h N_k \int_x \int_\theta (F_1 U_{1k} + F_2 U_{2k} + F_3 U_{3k}) a \, \mathrm{d}x \, \mathrm{d}\theta, \tag{3}$$

$$N_k = \int_x \int_\theta (U_{1k}^2 + U_{2k}^2 + U_{3k}^2) a \, \mathrm{d}x \, \mathrm{d}\theta. \tag{4}$$

The input forcing functions, F_1, F_2, F_3, in equation (3) are the forces applied at the joint in the axial, circumferential and transverse normal directions to the shell surface, and are functions of the co-ordinates x and θ. The displacement components, U_{1k}, U_{2k}, U_{3k}, for the mode k of the clamped–free shell without a plate in the x, y, and z directions can be represented with a beam function and its derivative, as below:

$$U_{1k}(x, \theta) = A\phi'(x) \cos n\theta, \qquad U_{2k}(x, \theta) = B\phi(x) \sin n\theta, \qquad U_{3k}(x, \theta) = C\phi(x) \cos n\theta. \tag{6–8}$$

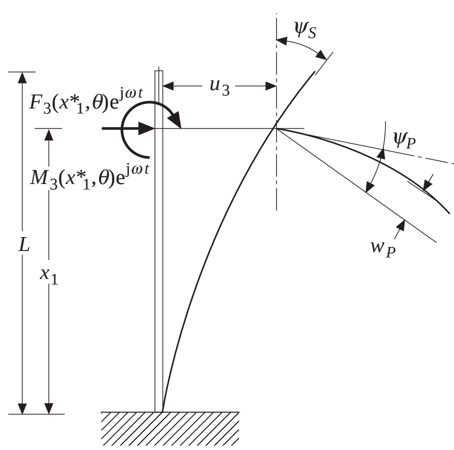

Figure 2. Displacement and slope due to dynamic loading at junction.

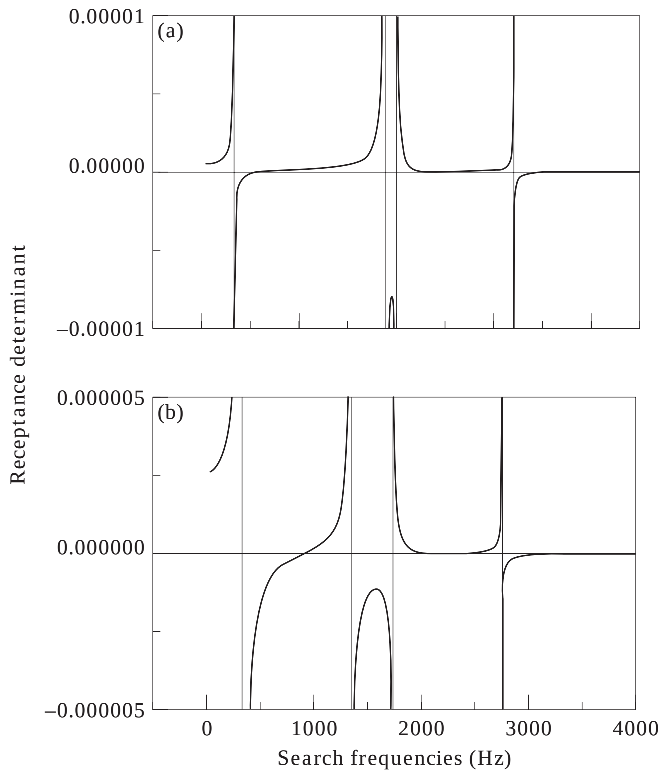

Figure 3. Frequency determinant for the circumferential wave number (a) $N = 0$ and (b) $N = 1$ of the shell with a plate at top.

where A, B, C are arbitrary constants to be determined, $\phi(x)$ is a beam function that satisfies the clamped–free boundary condition and $\phi'(x)$ is the derivative of the function below [17]:

$$\phi(x) = \cosh p_r x - \cos p_r x - C_r(\sinh p_r x - \sin p_r x), \tag{9}$$

$$\phi'(x) = \sinh p_r x + \sin p_r x - C_r(\cosh p_r x + \cos p_r x). \tag{10}$$

Table 1

Natural frequencies (Hz) from analysis, ANSYS and test for shell with an end cap

	$M = 1$			$M = 2$		
N	Analysis	ANSYS	Experiment	Analysis	ANSYS	Experiment
0	589	609	400	2360	2235	2125
1	603	575	425	1365	1271	1088
2	918	904	812	2103	2079	2100
3	739	728	700	1493	1464	1413
4	1060	1059	1050	1400	1400	1388
5	1636	1637	1637	1802	1813	1812
6	2371	2373	–	2477	2494	2500
7	3249	3248	–	3344	3349	–

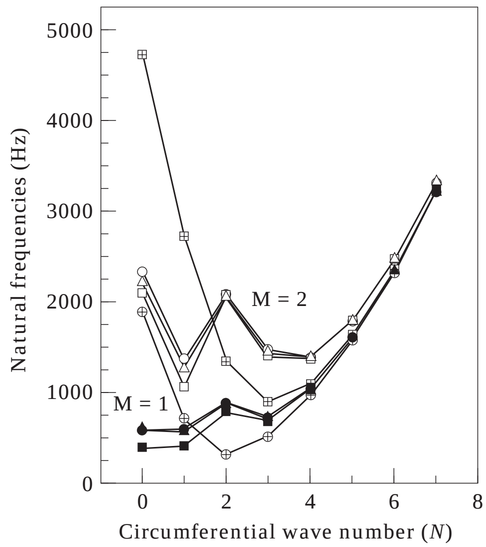

Figure 4. Frequencies as functions of circumferential wave number for the shell with a plate at top. Key: ($M = 1$), —■—, experimental; —●—, analysis; —▲—, ANSYS; —⊕—, C-F shell; ($M = 2$) —□—, experimental; —○—, analysis; —△—, ANSYS; —⊞—, C-F shell.

By applying the boundary condition in the beam function, C_r and p_r can be obtained from the following equations:

$$C_r = (\sinh p_r L - \sin p_r L)/(\cosh p_r + \cos p_r L), \qquad \cosh p_r L \cdot \cos p_r L + 1 = 0 \tag{11, 12}$$

From equation (2), the mode participation factor of mode k can be obtained as below:

$$\eta_k = \frac{f_k}{\omega_k^2 \sqrt{([1-(\omega/\omega_k)^2]^2 + 4\zeta_k^2(\omega/\omega_k)^2)}}\, e^{j\omega t}. \tag{13}$$

By neglecting the damping of the system, the displacements of the shell, u_i in equation (1) can be rewritten as:

$$u_i(x, \theta, t) = \sum_{k=1}^{\infty} \frac{f_k}{(\omega_k^2 - \omega^2)} U_{ik}(x, \theta)\, e^{j\omega t}. \tag{14}$$

Thus the displacements of the shell in each direction by the external forcing function, $f_k\, e^{j\omega t}$ can be represented by the mode summation and displacement components of the shell and will be used later to calculate the receptances.

By neglecting the other components of the displacements, except the transverse normal displacement of the shell, the only dynamic excitation to be considered at the junction ($x = x^*$) will be such a form as equation (15):

$$F_3(x^*, \theta) = \hat{F}_3 \cos n\theta\, \delta(x - x^*). \tag{15}$$

If one lets

$$\text{Denom} = (A/C)^2_{imn} \int_0^L \phi'^2(x)\, dx + (B/C)^2_{imn} \int_0^L \phi^2(x)\, dx + \int_0^L \phi^2(x)\, dx, \tag{16}$$

where i denotes the three roots of the frequency equation for the clamped–free shell and m and n are the numbers of the axial half waves and circumferential waves of the shell respectively, then the magnitude of the forcing function in equation (3) can be obtained as

$$f_k = \hat{F}_3 \phi(x^*)/\rho h \text{Denom} \tag{17}$$

Evaluation of equation (4) yields

$$N_{imn} = \pi a \text{Denom}, \qquad \text{for} \quad n \neq 0. \tag{18}$$

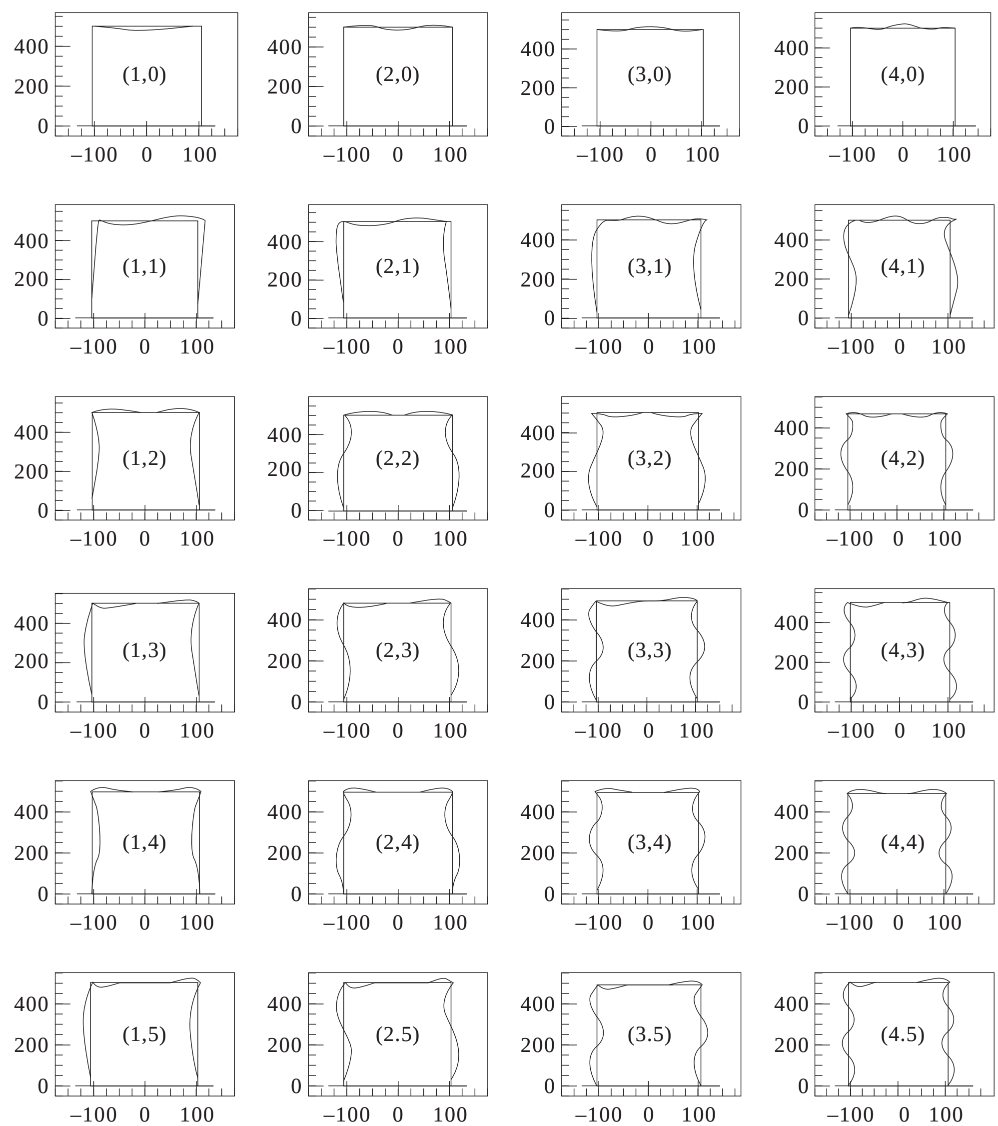

Figure 5. Mode shapes of the shell with a plate at top.

TABLE 2

Natural frequencies (Hz) from analysis, ANSYS and test for shell with a plate at its middle

	$M = 1$			$M = 2$		
N	Analysis	ANSYS	Experiment	Analysis	ANSYS	Experiment
0	585	645	(540)*	2102	2361	–
1	708	646	(475)	1372	1347	–
2	599	598	600	2274	2209	2137
3	615	619	620	1791	1688	–
4	1019	1026	1025	1566	1541	1474
5	1614	1621	1615	1889	1889	1850
6	2355	2360	2350	2535	2536	2500
7	3234	3237	3234	3379	3374	3475

* Note: ( ) means the mode shape was not confirmed.

Using equation (17) and equation (14), the dynamic displacement of the shell can be represented using the mode summation as

$$u_3(x, \theta, t) = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} \frac{1}{(\omega_{imn}^2 - \omega^2)} \frac{[\hat{F}_3 \phi(x^*) \phi(x)] \cos n\theta \, e^{j\omega t}}{\rho h \mathrm{Denom}}. \tag{19}$$

The slope of the shell in the axial direction can be obtained from equation (19) by differentiation with respect to the axial co-ordinate x.

$$\psi_S(x, \theta, t) = -\sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} \frac{1}{(\omega_{imn}^2 - \omega^2)} \frac{[\hat{F}_3 \phi(x^*) p_r \phi'(x)] \cos n\theta \, e^{j\omega t}}{\rho h \mathrm{Denom}}. \tag{20}$$

Next, the dynamic loading exerted at the junction ($x = x^*$) due to the constraint of the motion of the shell by the plate would be three directional components of the moments.

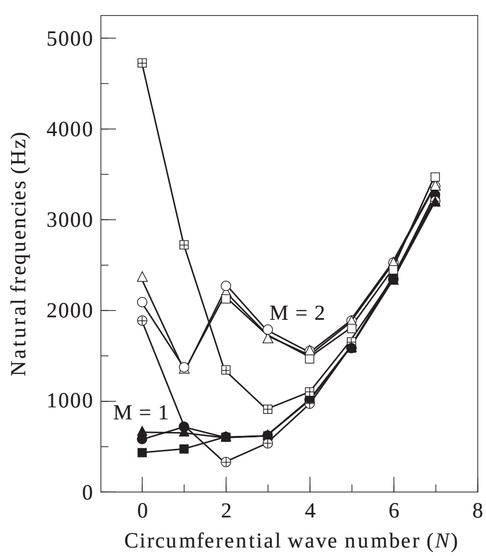

Figure 6. Frequencies as functions of circumferential wave number for the shell with a plate at middle.

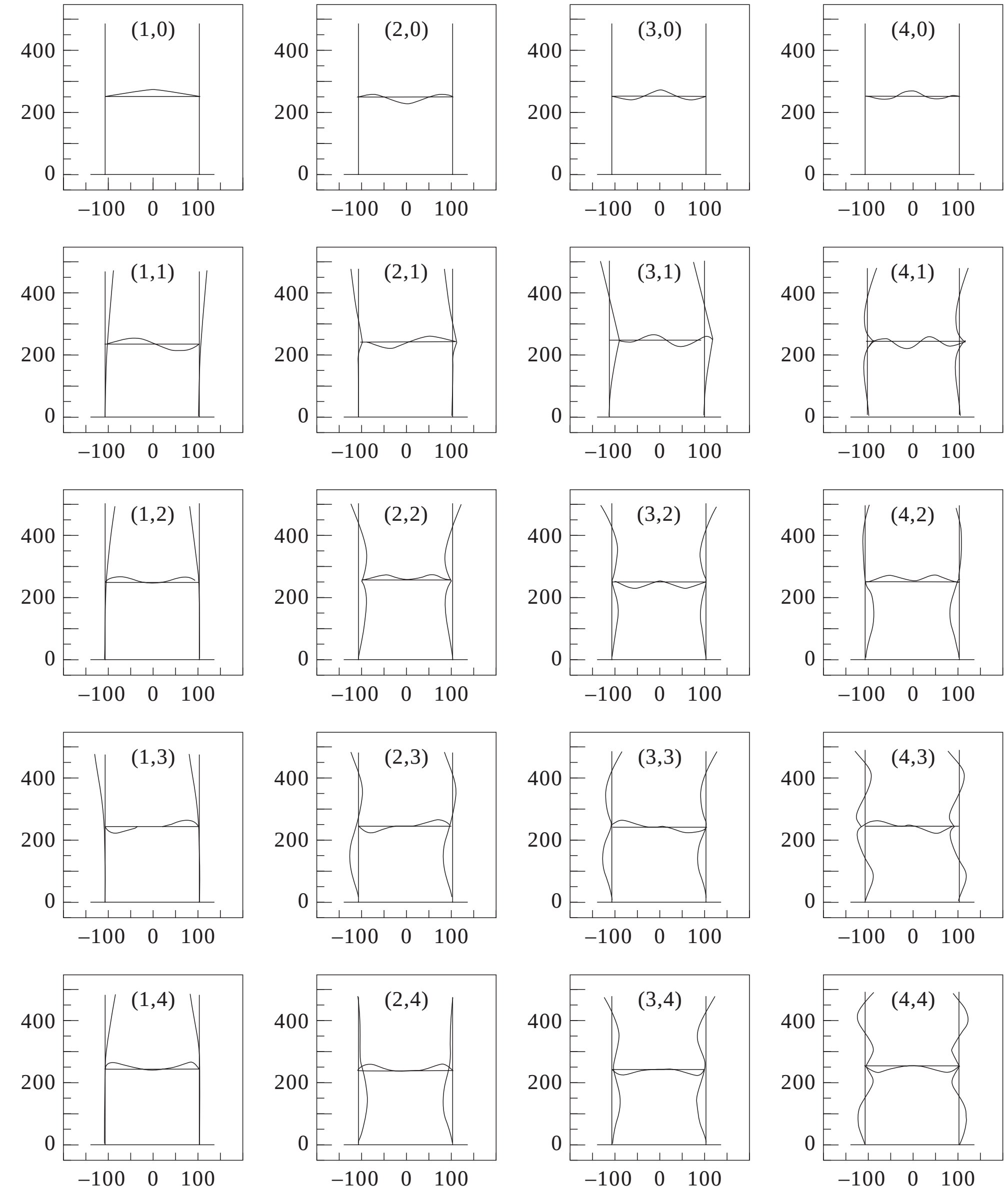

Figure 7. Mode shapes of the shell with a plate at middle. Key as for Figure 4.

Among them, small vibration displacement, the only moment M_3 that produces a transverse displacement of the shell and the plate is

$$M_3(x^*, \theta)\, e^{j\omega t} = \hat{M}_3 \cos n\theta\, \delta(x - x^*)\, e^{j\omega t}, \tag{21}$$

where δ is the Dirac delta function. With this moment loading, the forcing function of equation (3) can be evaluated from reference [18] as

$$f_k = 1/\rho h N_k \int_x \int_\theta U_{3k} \left[\frac{1}{a} \left(\frac{\partial (M_3(x^*, \theta))}{\partial x} \right) \right] a \, dx \, d\theta. \tag{22}$$

Here, N_k is the same as in equation (18). Neglecting the damping of the structure, the displacement of the shell due to the moment at the junction can be obtained from equations (14) and (22)

$$u_3(x, \theta, t) = -\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\sum_{i=1}^{3}\frac{\hat{M}_3}{(\omega_{imn}^2-\omega^2)}\frac{p_r\phi'(x^*)\phi(x)\cos n\theta\, \mathrm{e}^{\mathrm{j}\omega t}}{\rho h \mathrm{Denom}}. \tag{23}$$

Using equation (23), the slope of the shell by the moment at the junction can be calculated as

$$\psi_S(x, \theta, t) = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\sum_{i=1}^{3}\frac{\hat{M}_3}{(\omega_{imn}^2-\omega^2)}\frac{p_r^2\phi'(x^*)\phi'(x)\cos n\theta\, \mathrm{e}^{\mathrm{j}\omega t}}{\rho h \mathrm{Denom}} \tag{24}$$

The dynamic transverse loading and moment loading at the junction yield the translational rigid displacement u_P and the slope of the plate ψ_P. The final results, which are the same as those obtained by Huang and Soedel [10], can be rewritten as

$$u_P = -\frac{\hat{F}_3\cos\theta\, \mathrm{e}^{\mathrm{j}\omega t}}{a\rho h_P\omega^2}, \tag{25}$$

$$\psi_P(a, \theta, t) = -\sum_{m=1}^{\infty}\frac{\lambda^2 a\pi\hat{M}_3}{\rho h_P N_{mn}(\omega_{mn}^2-\omega^2)}\left[J_{n+1}(\lambda a)-\frac{J_n(\lambda a)}{I_n(\lambda a)}I_{n+1}(\lambda a)\right]^2\cos n\theta\, \mathrm{e}^{\mathrm{i}\omega t} \tag{26}$$

2.2. FREQUENCY EQUATION FOR THE SHELL WITH A PLATE

When a plate is attached at an arbitrary axial position of a shell, the frequency equation can be derived by considering the continuity condition at the shell/plate joining point. By applying the continuity condition at the junction,

$$\begin{bmatrix}\alpha_{11}+\beta_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}+\beta_{22}\end{bmatrix}\begin{Bmatrix}\hat{F}_3\\ \hat{M}_3\end{Bmatrix} = 0 \tag{27}$$

To get a non-trivial solution for equation (27), the frequency equation of the combined system can be derived as

$$\begin{bmatrix}\alpha_{11}+\beta_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}+\beta_{22}\end{bmatrix} = 0 \tag{28}$$

where α_{ij}, β_{ij} are the receptances of the shell and the plate, respectively. They are:

$$\alpha_{11} = \sum_{m=1}^{\infty}\sum_{i=1}^{3}\frac{1}{(\omega_{imn}^2-\omega^2)}\frac{\phi(x^*)\phi(x^*)}{\rho h \mathrm{Denom}}, \qquad \alpha_{21} = -\sum_{m=1}^{\infty}\sum_{i=1}^{3}\frac{1}{(\omega_{imn}^2-\omega^2)}\frac{p_r\phi(x^*)\phi'(x^*)}{\rho h \mathrm{Denom}}, \tag{29, 30}$$

$$\alpha_{22} = -\sum_{m=1}^{\infty}\sum_{i=1}^{3}\frac{1}{(\omega_{imn}^2-\omega^2)}\frac{p_r^2\phi'(x^*)\phi'(x^*)}{\rho h \mathrm{Denom}}, \tag{31}$$

$$\alpha_{12} = -\sum_{m=1}^{\infty}\sum_{i=1}^{3}\frac{1}{(\omega_{imn}^2-\omega^2)}\frac{p_r\phi(x^*)\phi'(x^*)}{\rho h \mathrm{Denom}}. \tag{32}$$

The receptances of the plate are the following forms [10]:

$$\beta_{11} = \begin{Bmatrix} 1/a\rho h_P\omega^2 & \text{for} & n=1 \\ 0 & \text{for} & n \neq 1 \end{Bmatrix}, \tag{33}$$

$$\beta_{22} = \sum_{m=1}^{\infty}\frac{\pi\lambda^2 a}{\rho h_P}\frac{[J_{n+1}(\lambda a)-[J_n(\lambda a)/I_n(\lambda a)]I_{n+1}(\lambda a)]^2}{N_{mn}(\omega_{mn}^2-\omega^2)}. \tag{34}$$

Here,

$$N_{mn} = \pi\int_0^{r=a}\left[J_n(\lambda r)-\frac{J_n(\lambda a)}{I_n(\lambda a)}I_n(\lambda r)\right]^2 r\,\mathrm{d}r. \tag{35}$$

Using equation (27), the moment to the lateral force ratio can be calculated and consequently, the mode shapes of the plate and shell will be:

$$w_P(r,\theta) = -\hat{F}_3\sum_{m=1}^{\infty}\frac{(\hat{M}_3/\hat{F}_3)\pi\lambda a}{\omega_{mn}^2-\omega^2}\frac{[J_n(\lambda r)-(J_n(\lambda a)/I_n(\lambda a))I_n(\lambda r)]}{\rho h_P N_{mn}}$$
$$\times[J_{n+1}(\lambda a)-(J_n(\lambda a)/I_n(\lambda a))I_{n+1}(\lambda a)]\cos n\theta, \tag{36}$$

$$u_3(x,\theta,t) = \sum_{m=1}^{\infty}\sum_{i=1}^{3}\frac{\hat{F}_3\phi(x)}{(\omega_{imn}^2-\omega^2)}\frac{\{\phi(x^*)-(\hat{M}_3/\hat{F}_3)p_r\phi'(x^*)\}\cos n\theta}{\rho h\mathrm{Denom}}. \tag{37}$$

2.3. FREQUENCY EQUATION FOR THE SHELL WITH TWO PLATE ATTACHMENTS

If two plates are attached at arbitrary axial positions of $x = x_1$ and $x = x_2$ separately, the continuity condition will result in

$$\begin{bmatrix} \alpha_{11}+\beta_{11} & \alpha_{12}+\beta_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21}+\beta_{21} & \alpha_{22}+\beta_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33}+\beta_{33} & \alpha_{34}+\beta_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43}+\beta_{43} & \alpha_{44}+\beta_{44} \end{bmatrix}\begin{Bmatrix} \hat{F}_{3,X1} \\ \hat{M}_{3,X1} \\ \hat{F}_{3,X2} \\ \hat{M}_{3,X2} \end{Bmatrix} = 0. \tag{38}$$

From equation (38), the frequency equation for the combined system will be equation (39).

$$\begin{vmatrix} \alpha_{11}+\beta_{11} & \alpha_{12}+\beta_{12} & \alpha_{13} & \alpha_{14} \\ \alpha_{21}+\beta_{21} & \alpha_{22}+\beta_{22} & \alpha_{23} & \alpha_{24} \\ \alpha_{31} & \alpha_{32} & \alpha_{33}+\beta_{33} & \alpha_{34}+\beta_{34} \\ \alpha_{41} & \alpha_{42} & \alpha_{43}+\beta_{43} & \alpha_{44}+\beta_{44} \end{vmatrix} = 0, \tag{39}$$

where the plate receptances β_{ij} are the same as in the equations (33), (34) and α_{ij} are the receptances of shell similar forms to equations (29–32).

3. NUMERICAL RESULTS AND DISCUSSIONS

The theoretical formulations were programmed to be executed by Lahey Fortran [19]. Individual receptances of the plate and shell were calculated and using these values, the frequencies of the system were obtained for each mode using the numerical method. Figure 3 shows examples of the frequency determinant (equation 28) as a function of the system frequency for the shell with a plate attached at its top. The frequencies to be determined are the zeros of the frequency determinant. The incremental root search finding and bisection method was employed to get more accurate results and fast iteration. The mode shapes were constructed with the calculated frequencies. Convergence was examined by comparing the results as a function of the mode summation in the displacement expression. The outer normal displacement of the shell, u_3, was considered to have converged if the displacement was represented by a mode summation of more than 30 modes of the axial half wave.

The material properties and dimensions of the shell and plate used were: length of the shell, $L = 500$ mm, radius of the shell and plate, $a = 104{\cdot}5$ mm, thickness of the shell or plate, $h_s = h_P = 3$ mm, Young's modulus, $E = 20{\cdot}6 \times 10^4$ N/mm^2, density of the material, $\rho_p = \rho_s = 7{\cdot}85 \times 10^{-9}$ Ns^2/mm^4 and Poisson's ratio, $\nu = 0{\cdot}3$.

3.1. NATURAL FREQUENCIES OF CLAMPED–FREE SHELL WITH AN END CAP

The frequencies obtained from the analysis as well as ANSYS and the test are listed in Table 1. Frequency variation as a function of the circumferential wave number for axial half wave 1 and two ($M = 1$ and 2) is plotted in Figure 4 to show more clearly the characteristics of the frequencies for the combined system. In Figure 4, the analysis results show good agreement with those of ANSYS, while the frequencies from the test are always lower than the results from the analysis or ANSYS, especially in the lower circumferential wave range. One of the reasons for these lower values seems to be the boundary effect. The analysis or ANSYS calculation uses the perfect clamped condition at the bottom of the shell and shell/plate welding. However, it is hard to realize the perfect clamping condition in the test due to the efficiency of the bottom clamping of the shell and the welding of the plate on the shell. It is also shown that at higher circumferential wave numbers, the frequencies do not differ much from one another.

The frequencies at the circumferential wave number 0 of a cantilevered shell with a plate become greatly diminished compared to those of a shell without a plate. This is because the system frequencies of these modes are highly dependent on the frequencies of the plate owing to the plate dominant modes, as in Figure 5. The lowest frequencies of the plate for the simple and clamped boundaries are 335 Hz and 692 Hz respectively. Thus, the first frequency of the combined system appeared between the two boundary conditions of the plate. The frequencyof the combined system at the circumferential wave number 1 with an axial half wave 1, i.e., mode (1, 1), seems not to be influenced by the plate because the mode of the combined system is a shell swaying mode and the plate does not affect the frequency of this mode of vibration of the shell. However, the frequencies of the combined system for the mode (1, 2) have a much bigger increase compared to the frequency of the shell without an end cap. It can be explained that this mode is a shell/plate combination mode and the shell oval motion is constrained to a great extent by the plate. Thus, the fundamental frequency of the shell without an end cap is no longer the fundamental frequency. The mode shapes in Figure 5 are exaggerated to show modes more clearly. The effect of the plate on the combined frequencies becomes negligibly small for the higher circumferential wave numbers, mainly because of the high frequencies of the plate. The frequency change of the combined system at axial half wave 2 can also be interpreted based on the mode shapes.

3.2. NATURAL FREQUENCIES OF CLAMPED–FREE SHELL WITH A PLATE AT ITS MIDDLE

For the case of a shell with a plate in its middle, the frequencies from analysis, ANSYS and test are listed in Table 2 and are shown in Figure 6 as functions of the circumferential wave number. The frequencies are shown to be almost the same as the case of a shell with a plate at its top. In Table 2, the frequencies in the bracket are those of the mode not confirmed from the test because the space in the shell is too narrow to impact on the plate. The influence of the plate on the frequencies here again becomes insignificant as the circumferential wave number goes high. The mode shapes are plotted in Figure 7 and are also exaggerated to show the modes more clearly.

In Figure 6, the frequencies of the clamped–free shell without a plate are decreased significantly due to the plate attachment at the circumferential mode 0 and 1. Because the modes of the circumferential wave 0 for the combined system are plate dominant modes, as in Figure 7, the frequencies of the combined system are expected to be influenced by the frequencies of the plate. Because the mode (1, 1) is the swaying mode of the shell, the frequencies of this mode of the shell without a plate are less influenced. On the contrary, the mode of circumferential wave 2 is the shell oval mode and is somewhat increased due to the constraint of the shell oval motion by the plate. For higher numbers of circumferential waves, the effect of the plate on the frequency is shown to be negligible, as was the case of a shell with a plate at its top.

4. CONCLUSIONS

As an extension of the receptance method to the clamped–free circular cylindrical shell with a plate attached in the shell at an arbitrary axial position, analytic formulations and their results were presented. A beam function was assumed to be the displacement function of the shell in the axial direction. Using the results of the analysis of free vibration of a clamped–free cylindrical shell by the Rayleigh–Ritz method and that of the circular plate, a frequency equation of the shell/plate combined system was obtained. Solving the frequency equation by the numerical method, the natural frequencies were obtained and the mode shapes were calculated. The frequencies and mode shapes were compared with those results from the finite element code, ANSYS and the free vibration test in order to validate the formulation. From the comparison, it was concluded that:

1) The formulations and procedure used herein could be applicable to the clamped–free shell with a plate attached at an arbitrary axial position.

2) The frequencies of the system were highly dependent on the frequencies of the dominant modes of the component.

3) The system frequencies for the higher range of circumferential wave numbers are not much influenced by the plate.

REFERENCES

1. C. B. SHARMA and D. J. JOHNS 1970 *TT*7001, *Department of Transport Technology*, *Loughborough University of Technology*. Vibration characteristics of clamped/free and clamped/ring stiffened circular cylindrical shells: a theoretical analysis.
2. C. B. SHARMA and D. J. JOHNS 1971 *Journal of Sound and Vibration* **14,** 459–474. Vibration characteristics of a clamped–free and clamped–ring stiffened circular cylindrical shell.

3. C. B. SHARMA and D. J. JOHNS 1972 *Journal of Sound and Vibration* **25,** 433–449. Free vibration of cantilever circular cylindrical shells—a comparative study.
4. C. B. SHARMA 1974 *Journal of Sound and Vibration* **35,** 55–76. Calculation of natural frequencies of fixed–free circular cylindrical shells.
5. C. B. SHARMA 1984 *Thin Walled Structures* **2,** 175–193. Free Vibration of clamped–free circular cylinders.
6. H. CHUNG 1981 *Nuclear Engineering and Design* **63,** 55–80. Analysis method for calculating vibrations characteristics of beams with intermediate supports.
7. S. WOLFRAM 1992 *Mathematica: A System for Doing Mathematics by Computer*. Massachusetts: Addison-Wesley. Second edition.
8. J. S. YIM, Y. S. LEE and D. S. SOHN 1996 *Advances in Computational Method for Simulation*, *Civil Company Press*, *Edinburgh*, 247–262. Free vibration analysis of clamped–free and clamped–free with ring stiffened circular cylindrical shells.
9. I. E. SHKAROV 1962 *NASA Technical Translation F*-341, 797–805. Use of the method of dynamic rigidities for calculating the frequencies of natural vibrations of built-up shells.
10. D. T. HUANG and W. SOEDEL 1993 *Journal of Sound and Vibration* **162,** 403–427. Natural frequencies and modes of a circular plate welded to a circular cylindrical shell at arbitrary axial positions.
11. D. T. HUANG and W. SOEDEL 1993 *Journal of Sound and Vibration* **166,** 315–339. On the free vibration of multiple plates welded to a cylindrical shell with special attention to mode pairs.
12. K. SUZUKI, S. TAKAHASHI, E. ANZAI and T. KOSAWADA 1983 *Journal of the Japanese Mechanical Society* **49,** (438), 154–164.
13. T. IRIE, G. YAMADA and Y. MURAMOTO 1984 *Journal of Sound and Vibration* **95,** 31–39. Free vibration of joined conical-cylindrical shells.
14. M. S. TAVAKOLI and R. SINGH 1989 *Journal of Sound and Vibration* **100,** 97–123. Eigensolution of joined/hermitic shell structures using the state space method.
15. Swanson Analysis Systems Inc. 1992 *ANSYS User's manual*.
16. W. SOEDEL 1993 *Vibration of Shells and Plates*. Hong Kong: Marcel Dekker.
17. R. D. BLEVINS 1979 *Formulas for Natural Frequency and Mode Shape*. New York: Van Nostrand Reinhold.
18. W. SOEDEL 1976 *Journal of Sound and Vibration* **48,** 179–188. Shells and plates loaded by dynamic moments with special attention to rotating point moments.
19. Lahey Computers Inc. 1994 *Lahey Fortran User's manual*.

NOMENCLATURE

A, B, C	unknown coefficients of displacement in the x, r, θ directions of the shell
a	radius of the shell or plate
$F_3, \hat{F}_3$	transverse dynamic force, magnitude of the force at junction
f_i	forces at shell/plate junction at axial, circumferential and transverse normal to the surface ($i = 1, 2, 3$)
h, h_S, h_P	thickness of shell, plate
k	mode number
L	length of the shell
m	axial half wave number
$M_P, \hat{M}_P$	dynamic moment, magnitude of the moment at junction
n	circumferential mode number
p_r	eigenvalue of beam function
r	radial co-ordinate of the plate
t	time
w_P	transverse normal displacement of plate
u_3	transverse normal displacement of shell
u_i	displacement of shell in i direction ($i = 1, 2, 3$)
U_{ik}	displacement components of the shell in i direction for mode k ($i = 1, 2, 3$)
x_1	axial co-ordinates of shell where plates are welded
x_1^*	axial co-ordinates where plate attached at $x = x_1$ of the shell
α_{ij}	receptances of shell ($i, j = 1, 2, 3, 4$)
β_{ij}	receptances of plate ($i, j = 1, 2, 3, 4$)
ψ_P, ψ_S	slope of plate and shell at axial co-ordinates
δ	Dirac delta function

λ	eigenvalue of plate
η_k	mode participation factor of mode k
ζ	damping of structure
ϕ	beam function
θ	co-ordinates in the circumferential direction
ω, ω_k, ω_{imn}	natural frequency, of mode k, of *imn* mode